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Title:

High-Order Multivariable Transfer Function Curve Fitting: Algorithms, Sparse Matrix Methods and Experimental Results

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ABSTRACT

This paper develops a computational approach to multivariable frequency domain curve fitting, based on 2-norm minimization, The algorithm is specifically tailored to the identification of complex systems having a large number of parameters, and includes a sparse matrix method for reducing computation and memory requirements on large problems. The algorithm is also well-suited for identification of lightly damped systems such as flexible structures. The overall approach is successfully demonstrated on a high-order multivariable flexible structure experiment requiring the estimation of 780 parameters over a 100 Hertz bandwidth.

1. INTRODUCTION

Let $G(\xi)$ be a n_u-input/n_y-output transfer function matrix in the complex variable ξ, where ξ can be chosen as any complex variable of interest (e.g., the Laplace operator s, the shift operator Z–l , or the delta operator $\delta = (.z - 1)/T$, where T is the sampling period). In order to allow a frequency domain interpretation, the complex variable ξ is itself considered a function of a frequency variable ω (e.g., $\xi = j\omega, \xi = e^{-j\omega T}, \xi = (e^{j\omega T} - 1)/7'$, for ξ = s, z^{-1}, δ, respectively).

In this paper, the data $\mathcal{G}$ is assumed to be given by noisy values of the transfer function mat rix evaluated over a grid of frequency points,

$$\mathcal{G}(\omega_i) \approx G(\xi(\omega_i)) \; i = 1, \ldots, N \tag{1.1}$$

The goal in this paper is to find a transfer function estimate G which minimizes the 2-norm of the error between the estimate and the data, i.e.,

$$F = \sum_{i=1}^{N} \mathrm{w}^2(\omega_i) \left\| \mathcal{G}(\omega_i) - G(\xi(\omega_i)) \right\|_f^2 \tag{1.2}$$

where $\mathrm{w}(\mathrm{w}_i)$ is a specified weighting function of frequency, and the Frobenious norm is defined as,

$$||X||_f^2 = Tr\{X^* X\} \tag{1.3}$$

where " * " denotes complex conjugate transpose,

It is emphasized that the criteria (1 .2) represents an output error, and would be optimal if statistically white noise entered at the output of the plant. Furthermore, the inverse of the noise coloring profile (if known or estimated) can be used as a frequency weighting to recover statistical optimality in the more general colored noise case.

Several S1S0 transfer function curve fitters are presently available in the literature. For curve fitting in the Laplace s domain, an early least squares method can be found in the 1959 work of Levy (1959). An implicit high frequency emphasis which distorted Levy's results was later noted by Sanathanan and Koerner (1963) and an iterative method to remove it appears in their 1963 paper. For convenience, their scheme is denoted here as the SK iteration. Improvements to complex curve fitting came soon after by introducing orthogonal polynomial bases (cf., Vlach 1969). The present state-of-the-art on the S1S0 problem (as assessed by hardware implementation on the Hewlett Packard analyzer), appears to be the method of Adcock (1987), which combines a Chebyshev polynomial basis with the SK iteration. Related results for the S1S0 case can be found in Spanos (1991) (for Laplace operator s) and Bayard et. al. (1991) (for the shift operator z).

MIMO curve fitters are less plentiful in the literature. Two papers of note are by Lin and Wu (1982), and Dailey and Lukich (1987). Both of these papers contain algorithms for curve fitting in the Laplace s domain using an SK iteration. The Dailey and Lukich approach is somewhat more sophisticated since it incorporates a Chebyshev polynomial basis, and is essentially a multivariable version of Adcock's approach. In the most recent literature, robust identification methods have appeared which are applicable to frequency domain data (see Gu and Khargonckar (1992) for overview and most general formulation), Numerical examples indicate that such methods can give excellent results for plants with moderate damping, However, the main drawback is that worst-case bounds become unrealistically large for lightly damped systems. Some practical modifications for lightly damped systems have been proposed in Gu and Khargonckar (1993). However, further research remains to be done in this area.

In contrast to the approaches mentioned above, the present paper performs MIMO frequency domain identification by minimizing the 2-norm of the curve fit error (1.2). This approach is specifically focused to support the identification of high order systems with many parameters, and can provide very good results for lightly damped systems (cf., Bayard 1992c, 1993). The algorithm is depicted in Fig. 1, and is based on a Gauss-Newton (GN) iteration, initialized using the SK iteration. Both iterations require solving least squares problems which are formulated to avoid solving normal equations. A sparse matrix QR (or sparse matrix SVD) technique is developed which exploits the special block structure of the problem and significantly reduces memory and computational requirements. The overall approach is demonstrated on a real experimental data set involving a multivariable flexible structure.

2. BACKGROUND AND NOTATION

The transfer function matrix $G(\xi)$ is considered to be in the form of the ratio of a matrix numerator polynomial $N(\xi)$ and an nth-order monic scalar denominator polynomial $d(\xi)$, i.e.,

$$G(\xi) = \frac{N(\xi)}{d(\xi)} \tag{2.1}$$

In order to allow the most flexibility in the approach, d and N are each written in terms of polynomial basis $p^k(\xi)\, k = 0,\ldots, n$, as follows,

$$N(\xi) = \sum_{k=0}^{n_b} B_k p^k(\xi) \tag{2.2a}$$

$$d(\xi) = 1 + \xi a(\xi) \tag{2.2b}$$

$$a(\xi) = \sum_{k=1}^{n} a_k p^{k-1}(\xi) \tag{2.2c}$$

Here, $B_k \in R^{n_y \times n_u}$, $k = O, \ldots . n_b$ where n_b equals n or $n-1$ depending on whether or not a feedforward term exists on the plant G. The terms $p^k(\xi)$, $k = O, \ldots . n$ are polynomials of degree k in the complex variable ξ which can be chosen to improve the numerical properties of the algorithm (see Sect. 6.).

The goal is to find a transfer function estimate G which minimizes the 2-norm of the error (1.2) between the estimate and the data. The cost F can be expanded by substituting (2.1) and (2.2) into (1.2) to give upon rearranging,

$$F \sum_{i=1}^{N} \frac{w^2(\omega_i)}{|1 + \xi(\omega_i)a(\xi(\omega_i))|^2} \left\| \mathcal{G}(\omega_i) + \xi(\omega_i)\mathcal{G}(\omega_i)a(\xi(\omega_i)) \sum_{k=O}^{n_b} B_k p^k(\xi(\omega_i)) \right\|_f^2 \tag{2.3}$$

Since the Frobenious norm of a matrix is the sum-squared magnitudes of its entries, expression (2.3) can written in terms of its components as follows,

$$\sum_{\ell=1}^{n_y} \sum_{m=1}^{n_u} \sum_{i=1}^{N} \frac{w(\omega_i)^2}{|1 + 2r(\omega_i)a + a^T R(\omega_i)a|} \left| \mathcal{G}^{\ell m}(\omega_i) - \phi^{\ell m}(\omega_i)a - \psi(\omega_i)b^{\ell m} \right|^2 \tag{2,4}$$

where,

$$\mathcal{G}(\omega_i) = \{\mathcal{G}^{\ell m}(\omega_i)\} \tag{2.5a}$$

$$a = [a_1, a_2, \ldots, a_n]^T \tag{2.5b}$$

$$b^{\ell m} = [b_0^{\ell m}, b_1^{\ell m}, \ldots, b_{n_b}^{\ell m}]^T \tag{2.5c}$$

$$B_k = \{b_k^{\ell m}\}; \quad k = O, \ldots, n_b \tag{2.5d}$$

$$\phi^{\ell m}(\omega_i) = -\xi(\omega_i)[p^0(\xi(\omega_i)), p^1(\xi(\omega_i)), \ldots, p^{n-1}(\xi(\omega_i))]\mathcal{G}^{\ell m}(\xi(\omega_i)) \tag{2.5e}$$

$$\eta(\omega_i) = \xi(\omega_i)[p^0(\xi(\omega_i)), p^1(\xi(\omega_i)), \ldots, p^{n-1}(\xi(\omega_i))] \tag{2.5f}$$

$$r(\omega_i) = \Re\{\eta(\omega_i)\} \tag{2.5g}$$

$$R(\omega_i) = \Re\{\eta^*(\omega_i)\eta(\omega_i)\} \tag{2.5h}$$

$$\psi(\omega_i) = [p^0(\xi(\omega_i)), p^1(\xi(\omega_i)), \ldots, p^{n-1}(\xi(\omega_i))] \tag{2.5i}$$

$$for\ \ell = 1, \ldots,\ n_y;\ m\ =\ 1, \ldots,\ n_u;\ \ i\ = 1, \ldots, N$$

The expression (2,4) can be vectorized and written in terms of real arithmetic to give

$$F\ \ =\ \ W(a)(y\ -\ H\theta)\ \ {}_{2}^{2} \tag{2.6}$$

where $||\ ,\ ||_2$ denotes the vector 2-norm, and

$$W(a)\ \ = \begin{bmatrix} \overline{W}S(a) & O\,0\ .\ .\ . & 0 \\ 0 & \overline{W}S(a) & \ddots\ \ddots\ \ddots \\ \vdots & \ddots & 0 \\ 0 & \ldots & 0\ \ \overline{W}S(a) \end{bmatrix} \tag{2.7a}$$

$$\overline{W} = diag\{\mathrm{w}(\omega_1), \ldots, \mathrm{w}(\omega_N)\} \tag{2.7b}$$

$$S(a)\ =\ diag\left\{ \frac{1}{\sqrt{1 + 2r(\omega_1)a + a^T R(\omega_1)a}}, \ldots, \frac{1}{\sqrt{1 + 2r(\omega_N)a + a^T R(\omega_N)a}} \right\} \tag{2.7c}$$

$$H = \begin{bmatrix} \Psi & 0 & \ldots & 0 & \Phi^{11} \\ 0 & \Psi & \ddots & \vdots & \Phi^{12} \\ \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \ldots & 0 & \Psi & \Phi^{n_y n_u} \end{bmatrix} \tag{2.7d}$$

$$\Phi^{\ell m} = \begin{bmatrix} \Re\{\phi^{\ell m}\} \\ \Im\{\phi^{\ell m}\} \end{bmatrix}; \qquad \phi^{\ell m} = \begin{bmatrix} \phi^{\ell m}(\omega_1) \\ \phi^{\ell m}(\omega_2) \\ \vdots \\ \phi^{\ell m}(\omega_N) \end{bmatrix} \tag{2.7e}$$

$$\ell = 1, \ldots, n_y;\ m\ =\ 1, \ldots,\ n_u$$

$$\Psi = \begin{bmatrix} \Re\{\psi\} \\ \Im\{\psi\} \end{bmatrix}; \qquad \psi = \begin{bmatrix} \psi(\omega_1) \\ \psi(\omega_2) \\ \vdots \\ \psi(\omega_N) \end{bmatrix} \tag{2.7f}$$

$$\theta = [b^T,\ a^T]^T \tag{2.7g}$$

$$b\ =\ [(b^{11})^T, (b^{12})^T, \ldots, (b^{mn})^T]^T \tag{2.7h}$$

$$y = \begin{bmatrix} y^{11} \\ y^{12} \\ \vdots \\ y^{mn} \end{bmatrix}; \quad y^{\ell m} = \begin{bmatrix} \Re\{g^{\ell m}\} \\ \Im\{g^{\ell m}\} \end{bmatrix}; \quad g^{\ell m} = \begin{bmatrix} \mathcal{G}^{\ell m}(\omega_1) \\ \mathcal{G}^{\ell m}(\omega_2) \\ \vdots \\ \mathcal{G}^{\ell m}(\omega_N) \end{bmatrix} \tag{2.7i}$$

3. GAUSS-NEWTON ALGORITHM

While general unconstrained optimization methods can be used minimize F in (2.6), more efficient methods can be used which exploit the special structure of the problem. One such method is the Gauss-Newton (GN) algorithm (cf., Gill, Murray and Wright, 1981) which is applicable to nonlinear least squares problems of the form,

$$\min_{\theta} ||f(\theta)||_2^2 \tag{3.1}$$

The GN method is applied to the present problem by noting that *F in (2.6)* can be put into the form (3. 1) by the choice,

$$f(\theta) = W(a)(y - He) \tag{3.2}$$

The GN iteration is given by the following expression (Gill, Murry and Wright, 1981, page 134),

$$\theta^{k+1} = arg \min_{\theta} ||J(\theta^k)(\theta - \theta^k) + f(\theta^k)||_2^2 \tag{3.3}$$

where $J(\theta)$ is the Jacobian of $f(\theta)$. Using (3.2), the expression for the Jacobian can be computed as,

$$J(\theta) = -W(a)(H + \tilde{H}(\theta)) \tag{3.4}$$

where,

$$\tilde{H}(\theta) = \begin{bmatrix} 0 & \dots & 0 & \mathcal{A}^{11} \\ 0 & \dots & 0 & \mathcal{A}^{12} \\ \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & 0 & \mathcal{A}^{n_y n_u} \end{bmatrix} \tag{3.5a}$$

$$\mathcal{A}^{\ell m} = diag\{e^{\ell m}(\theta)\} \begin{bmatrix} r^T(\omega_1) + a^T R(\omega_1) \\ r^T(\omega_2) + a^T R(\omega_2) \\ \vdots \\ r^T(\omega_N) + a^T R(\omega_N) \end{bmatrix} \tag{3.5b}$$

$$e^{\ell m}(\theta) = S^2(a)\left(y^{\ell m} - \Psi b^{\ell m} - \Phi^{\ell m} a \right) \tag{3.5c}$$

Substituting (3.2) and (3.4) into the argument of (3.3) gives,

$$\begin{aligned} J(\theta^k)(\theta - \theta^k) + f(\theta^k) &= -W(a^k)\big(H + \tilde{H}(\theta^k)\big)(\theta - \theta^k) + W(a^k)(y - H\theta^k) \\ &= W(a^k)[\big(\tilde{H}(\theta^k)\theta^k + y\big) - \big(H + \tilde{H}(\theta^k)\big)\theta] \end{aligned} \tag{3.6}$$

Finally, substituting (3,6) into (3.3) gives the GN iteration in terms of the following sequence of reweighted of least squares problems,

Gauss-Newton Iteration:

$$\theta^{k+1} = arg \min_{\theta} \left\| W(a^k)\big(y_G(\theta^k) - H_G(\theta^k)\theta\big) \right\|_2^2 \tag{3.7}$$

where,

$$H_G(\theta^k) = (H + \tilde{H}(\theta^k)) \tag{3.8a}$$

$$y_G(\theta^k) = (\tilde{H}(\theta^k)\theta^k + y) \tag{3.8b}$$

A kcy observation is the sparse structure of the matrix H_G,

$$H_G(\theta) = \begin{bmatrix} \Psi & 0 & \dots & 0 & \Phi^{11} + \mathcal{A}^{11} \\ 0 & \Psi & \ddots & \vdots & \Phi^{12} + \mathcal{A}^{12} \\ \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & 0 & 0 & \Psi & \Phi^{n_y n_u} + \mathcal{A}^{n_y n_u} \end{bmatrix} \tag{3.9}$$

4. INITIALIZATION VIA SK ITERATION

Since the Gauss-Newton algorithm is a descent method, it will generally only converge to a local minimum of (2.6). Hence, the starting value for θ is crucial. A special algorithm is discussed in this section to provide a good starting value.

The method to bc used, is a multivariable generalization of the algorithm of Sanathanan and Koerner (1963) algorithm discussed in Bayard (1992 b). It is defined by the following "SK iteration" composed of a sequence of reweighted least squares problems,

SK Iteration:

$$\theta^{k+1} = arg \min_{\theta} \quad W(a^k)(y - H\theta)_2^2 \tag{4.1}$$

with initial condition,

$$\theta^0 = [(b^0)^T, (a^0)^T]^T = O \tag{4.2}$$

Remark 1. When the plant has lightly damped resonances, the weighting $W(a^k)$ in (4,1) tends to bc very peaked in the vicinity of the resonance frequencies. This can act as a "hole-punch" to essentially pick out only a small subset of data points, leaving an underdetermined system of equations, It was found in Bayard *et. al.* (1991) that smoothing $W(a^k)$ slightly as a function of frequency tends to offset this effect and improve the numerical conditioning of the problem. ■

Remark 2. A crucial observation relevent to identification of high-order systems is that the SK and GN iterations are both implemented here *without* solving normal *equations.* ■

Remark 3. The model order is generally not known a-priori, and in practice it is useful to try several different orders to obtain the best results. Alternatively, one can intentionally overparamet rize the model, The extra dynamics introduced by overparametrization can then be systematically removed (without polynomial factorization) using the SSFD algorithm given in Bayard (1992 c). Interestingly, it has been shown in Bayard (1993) that the extra dynamics from overparametrizing in the shift operator $\xi = z^{-1}$ tend to come in stably, while the dynamics from overparametrizing the s and δ operators tend to come in unstably. Modifications of the s and δ operators which overparametrize stably are given in Bayard (1993), but are beyond the scope of this discussion. ■

5. SPARSE MATRIX QR (AND SVD) METHOD

A key observation is that H in the SK iteration and H_G in the Gauss-Newton iteration, (i.e., (2.7d) and (3.9), respectively), have identical block structure i.e.,

$$H = \begin{bmatrix} \Psi & 0 & \cdots & 0 & X_1 \\ 0 & \Psi & \ddots & \vdots & X_2 \\ \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \cdots & 0 & \Psi & X_M \end{bmatrix} \tag{5.1}$$

Here $M = n_y n_u$, and the X_i are generic blocks, depending upon which algorithm is being used. Note that the index i on Xi runs sequentially as i = 1, ... M to avoid the notational complexities of double indexing used earlier in (2.7d) and (3.9).

The special block structure of (5.1) will be exploited in this section to develop a sparse matrix QR and SVD factorization to solve the least squares problems arising in both the SK and GN iterations. Towards this end, both least squares problems can be put into the general form,

$$\min \|y - H\theta\|_2^2 \tag{5.2}$$

where H has the block structure in (5.1), and vectors θ and y are partitioned compatibly with H as,

$$\theta = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_M \\ a \end{bmatrix}; \quad y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_M \end{bmatrix} \tag{5.3}$$

The least squares problem (5.2) can be solved using the following sequence of steps.

Sparse Matrix QR/Least Squares Algorithm:

Step 1: Perform QR factorization on Ψ to give,

$$\Psi = [Q_{\psi 1} \quad Q_{\psi 2}] \begin{bmatrix} R_\psi \\ 0 \end{bmatrix} \tag{5.4a}$$

Step *2:* Form $Q^\perp = I - Q_{\psi_1} Q_{\psi_1}^T$ and compute the quantity V where,

$$V = \begin{bmatrix} Q^\perp X_1 \\ Q^\perp X_2 \\ \vdots \\ Q^\perp X_M \end{bmatrix} \tag{5.4b}$$

Step *3:* Perform QR factorization on V to give,

$$V = [Q_{v_1} \quad Q_{v_2}] \begin{bmatrix} R_v \\ 0 \end{bmatrix} \tag{5.4c}$$

Step 4: Form z as follows,

$$z = Q_{v1}^T y \tag{5.4d}$$

and backsolve triangular system for a,

$$R_v a = z \tag{5.4e}$$

Step 5: For $j = 1, ..., M$:

$$z_j = Q_{\psi 1}^T \, y_j \tag{5.4f}$$

backsolve for b_j

$$R_\psi b_j = z_j - Q_{\psi 1}^T X_j a \tag{5.4g}$$

An explicit QR factorization of the sparse matrix H *is* given in the next theorem. Although the full factorization is never formed explicitly, this result serves as a proof of the above algorithm.

Theorem 1. Sparse *Matrix QR Factorization:*

Using the quantities computed in Steps 1-3 above, the QR factorization of the sparse matrix H with the block structure (5.1) is given by,

$$H = QR = [Q_1 \;\; Q_2] \begin{bmatrix} R_{11} & R_{12} \\ 0 & R_{22} \end{bmatrix} \tag{5.5}$$

where,

$$Q_1 = \begin{bmatrix} Q_{\psi 1} & 0 & \dots & 0 \\ 0 & Q_{\psi 1} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & Q_{\psi\,1} \end{bmatrix}; \quad Q_2 = Q_{v1} \tag{5.6a}$$

$$R_{11} = \begin{bmatrix} R_\psi & 0 & \dots & 0 \\ 0 & R_\psi & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & R_\psi \end{bmatrix} \tag{5.6b}$$

$$R_{12} = \begin{bmatrix} Q_{\psi 1}^T X_1 \\ Q_{\psi 1}^T X_2 \\ \vdots \\ Q_{\psi 1}^T X_M \end{bmatrix}; \; R_{22} = R_v \tag{5.6c}$$

Proof R is triangular by construction. It can be readily verified that $QR = H$, and $Q^T Q = I$ as desired. ∎

Since Q in (5.5) is not square, Theorem 1 provides a "skinny" QR factorization in the sense of Golub and Van Loan (1989). It is noted that only "skinny" QR factorization are required in Steps 1 and 3. of the sparse matrix QR algorithm i.e., the quantities Q_{v2} and $Q_{\psi 2}$ are not used. Hence their computation should be avoided to reduce storage and computational requirements.

Remark 4. Sparse *Matrix SVD Algorithm:* Theorem 1 (and the previous algorithm) can also bc used for sparse matrix singular value decomposition (SVD), by noting that the SVD decomposition $H = U\Sigma V^T$ is of the form $H = QR$ where Q = U is an orthogonal matrix and R is defined as $R = \Sigma V^T$. Of course, in this case R is no longer triangular and the backsubstitution in Steps 4 and 5, can be replaced by inversion of R using the relation $R^{-1} = V\Sigma^{-1}$. ■

6. IDENTIFICATION USING THE Z-l, s, and δ OPERATORS

Choices of polynomial basis are discussed in Bayard (1992b) for curve fitting in the z^{-1}, s, and δ operators. These choices are briefly outlined below.

z^{-1} - Operator

$$p^k(\xi) = \xi^k; \quad \xi = z^{-1} = e^{-j\bar{\omega}}; \quad \bar{\omega} = \omega T; \quad \bar{\omega} \in [0, \pi] \tag{6.1}$$

s - Operator

$$p^k(\xi) = \begin{cases} T_k(\xi) \text{ with signs flipped on powers } 2,6,10...; & \text{for k even,} \\ T_k(\xi) \text{ with signs flipped on powers } 3,7,11,..; & \text{for k odd} \end{cases} \tag{6.2}$$

$$\xi = s/\omega_{max} = j\bar{\omega}; \quad \bar{\omega} = \omega/\omega_{max}; \quad \bar{\omega} \in [0,1] \tag{6.3}$$

where the Chebychev polynomials are generated as $T_0(x) = 1$, $T_1(x) = x$ and $T_k(x) = 2xT_{k-1} - T_{k-2}$.

δ - Operator

$$\xi = \delta/\omega_{max} \frac{(e^{j\omega T} - 1)}{\omega_{max} T}; \quad \bar{\omega} = \omega/\omega_{max}; \quad \bar{\omega} \in [0,1] \tag{6.4}$$

For fast sampling $\xi \simeq j\bar{\omega}$. Hence powers of ξ behave like powers of $j\bar{\omega}$ and the basis can bc chosen based on the Chebyshev polynomials as given in (6.2).

70 EXPERIMENTAL STUDY

This example demonstrates the curve fitting algorithm (in the shift operator ξ = zI), on real experimental data taken from the JPL Advanced Reconfigurable Control Testbed shown in Fig, 2. A 4-input, 3-output transfer function is considered, where each actuator is an active strut, and each sensor is an accelerometer. The frequency response data is obtained using a 512 Schroeder phased sum-of-sinusoids input design at a sampling rate of 200 Hertz (see Bayard (1992a) for background on estimation with Schroeder-phased inputs). The magnitude response is shown as the dashed line in Fig. 3 (phase is available but not shown).

The model order is chosen as n = 60, and a uniform weighting $w(\omega_i) \equiv 1$ is used. Since there are 12 numerator polynomials and 1 denominator polynomial, this requires the simultaneous estimation of 780 parameters. The SK algorithm is iterated 12 times, and then the GN algorithm is iterated 3 times, both algorithms using the sparse matrix SVD met hod developed in Sect. 5 (cf., Remark 4). A magnitude plot of the identified model is shown in Fig. 3 (solid line) superimposed on the response data (dashed line). The model is stable, and is seen to match the data well in all channels, and over the full 100 Hertz bandwidth,

8. CONCLUSIONS

A multivariable curve fitter is proposed for frequency domain identification. The approach is based on 2-norm minimization, uses an SK iteration to initialize the GN algorithm, avoids solving normal equations, and utilizes a sparse matrix QR (or alternatively SVD) method for efficiently solving the underlying SK and GN iterations, The method is applicable to high-order multivariable systems and works well on lightly damped systems.

The overall approach was successfully demonstrated on real experimental data from a 4-input/3-output multivariable flexible structure requiring the simultaneous estimation of 780 parameters, and leading to a multivariable transfer function fitting the data accurately over a 100 Hertz bandwidth. The sparse matrix SVD method was indispensable for this problem, reducing RAM memory requirements by better than an order of magnitude (from approximately 60 Megabytes to 6 Megabytes) and reducing computation time two orders of magnitude (from approximately 30 hours to 20 minutes on a Spare 2 workstation).

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FIGURES

Figure 1. Algorithm for multivariable frequency domain identification

Figure 2. JPL Advanced Reconfigurable Control (ARC) testbed

Figure 3. Multivariable experimental identification results: Raw experimental frequency data (dashed); Identified 780 parameter, 4-input/3-output transfer function model $G(z^{-1}) = N(z^{-1})/d(z^{-1})$ where $n = 60$, (solid).

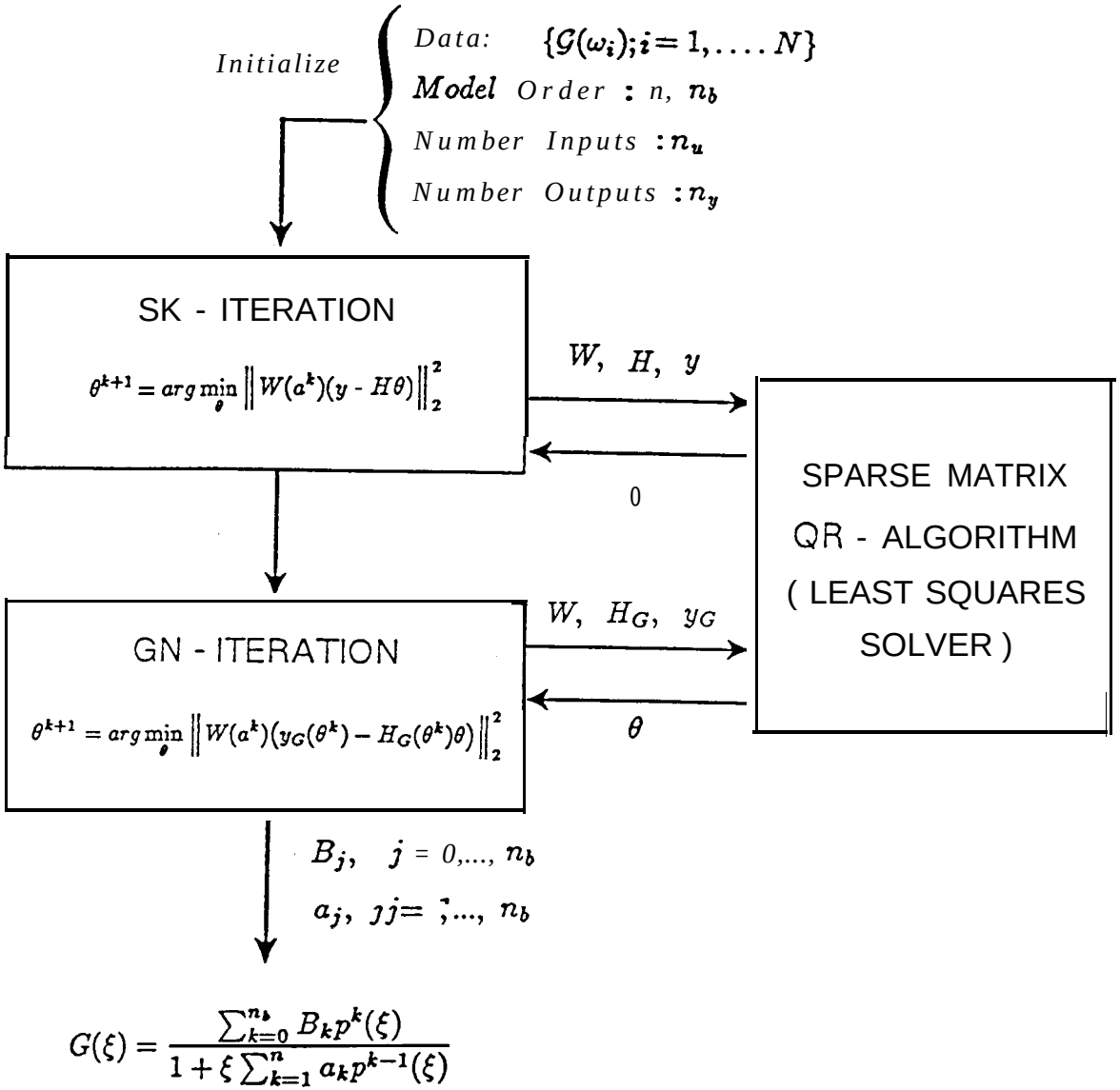
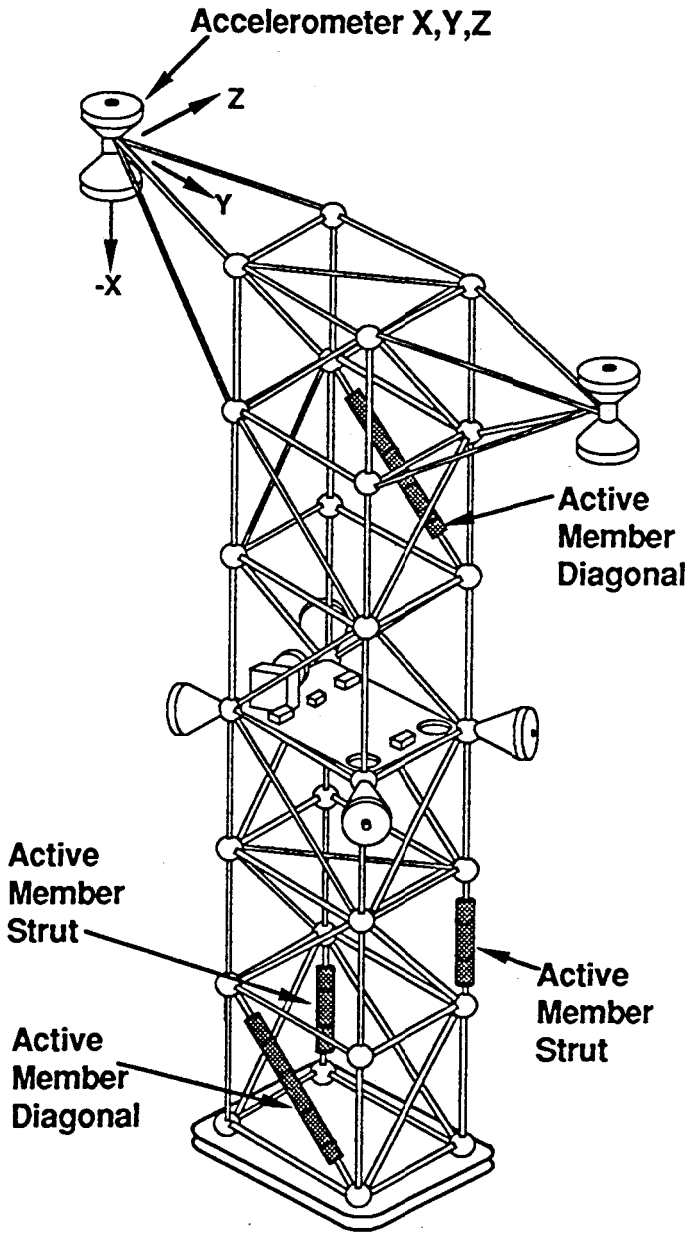


Figure 1. Algorithm for Multivariable Frequency Domain Identification

Fig. 1


Accelerometer X,Y,Z
Z
Y
-X
Active
Member
Diagonal
Active
Member
Strut
Active
Member
Strut
Active
Member
Diagonal

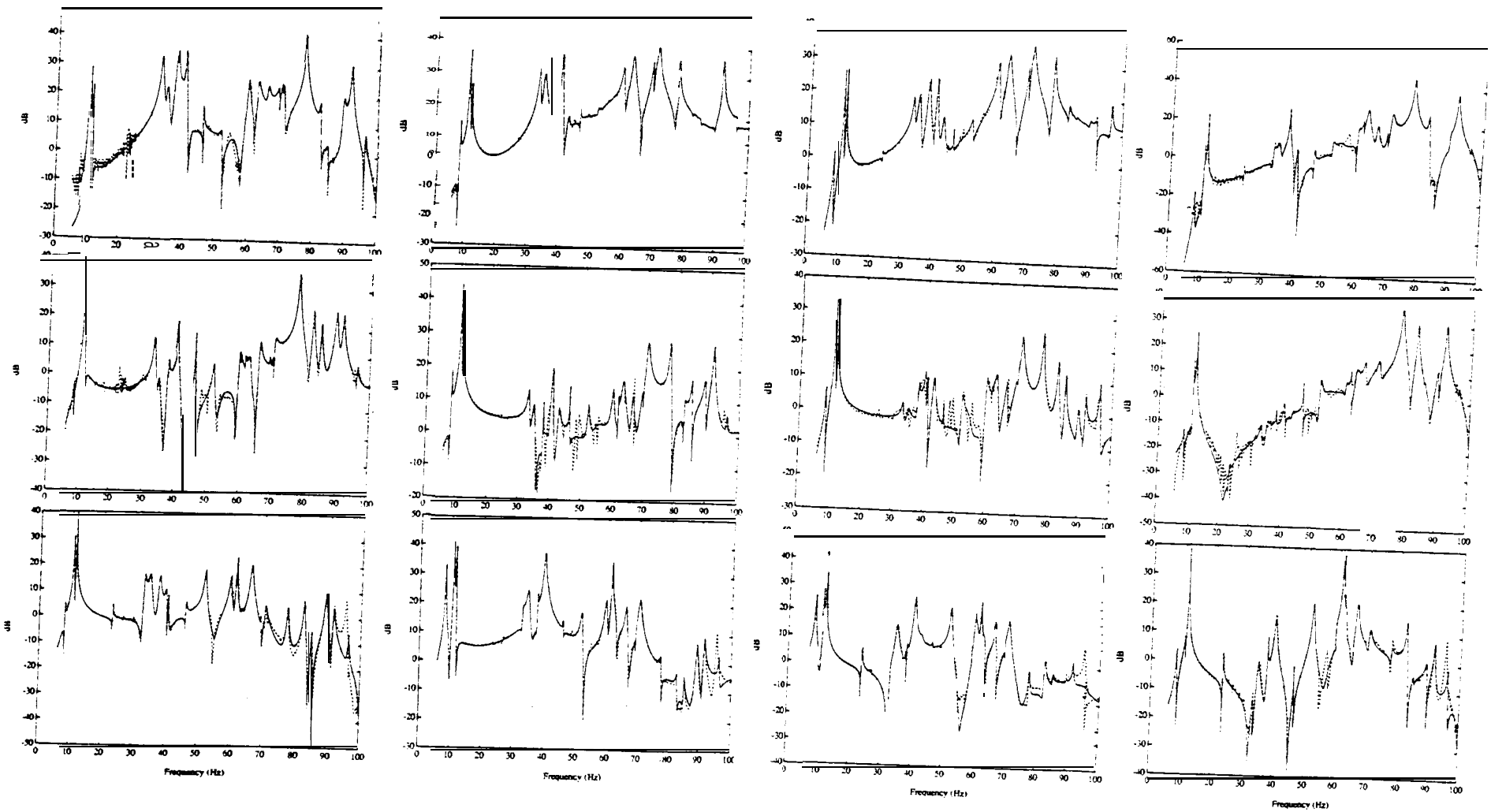


Fig. 3
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